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# Theoretical Calculations of the Masses of the Elementary Fermions

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## Abstract

Our universe is three-dimensional and curved (with a positive curvature) and thus may be embedded in a four-dimensional Euclidean space with coordinates  $x, y, z, t$  where the fourth dimension time  $t$  is treated as a regular dimension. One can set in this spacetime a four-dimensional underlying array of small hypercubes of one Planck length edge. With this array all elementary particles can be classified following that they are two-, three-, or four-dimensional. The elementary wavefunctions of this underlying array are equal to  $\sqrt{2} \exp(ix_i)$  for  $x_i = x, y, z$  or to  $\sqrt{2} \exp(it)$  for  $t$ . Hence, the masses of the fermions of the first family are equal to  $2^n$  (in  $eV/c^2$ ) where  $n$  is an integer. The other families of fermions are excited states of the fermions of the first family and thus have masses equal to  $2^n \cdot (p^2)/2$  where  $n$  and  $p$  are two integers. Theoretical and experimental masses fit within 10%.

**Keywords:** four-dimensional spacetime, masses of elementary fermions, theoretical masses, real space theory, Grand Unified Theory

## 1. Introduction

Since the beginning of the twentieth century, experimental particle physics has been making large progresses with the set up of accelerators and colliders.

The main locations of accelerators are presently: the Centre Europeen de Recherches Nucleaires (CERN) near Geneva (Switzerland and France). Equipments of the CERN are presently the Super Proton Synchrotron and the Large Hadron Collider (LHC), which is a protons collider. In Germany, the DESY (Deutsche Elektronen Synchrotron) main set up HERA is a collider between electrons or positrons and protons. In the USA, the Stanford Linear Accelerator Center (SLAC) main set up is PEP-II, which is a collider between electrons and positrons. Located also in the USA, the Fermi National Accelerator Laboratory (Fermilab) uses its main set up the Tevatron to collide protons and antiprotons. Finally, the Brookhaven National Laboratory (USA) uses the set up Relativistic Heavy Ion Collider to study collisions between heavy ions.

Up to now, the results obtained with colliders and accelerators fit the Standard Model, which predicts the existence of three families of elementary fermions and five different types of bosons. Although string theories [1] and supersymmetry [2] try to unify all different types of elementary particles, no experimental proof has

been made of these theories. So, we present here a new theory that aims to unify elementary particles characteristics. The theoretical masses of the elementary particles are compared to the experimental masses.

This book chapter is a small review of the theoretical calculations of the masses of elementary particles in real space [3–5]. The theoretical masses that we calculated fit the experimental masses within less than 10% for almost all elementary fermions.

Our universe is three-dimensional and has a positive curvature. So our universe may be embedded in an Euclidean four-dimensional space. These four dimensions are  $x, y, z, t$  where  $t$  is time [6–8]. In this four-dimensional space, we classify the elementary particles following their geometry, that is, elementary particles may be four-, three-, or two-dimensional [5] (see Section 2). Let us notice that for a given particle, time and mass are linked: if the mass of a particle is zero, this particle has no temporal dimension.

If spacetime is composed of small hypercubes of one Planck length edge, there exist elementary wavefunctions that are equal to  $\sqrt{2} \exp(ix_i)$  if it corresponds to a space dimension or equal to  $\sqrt{2} \exp(it)$  if it corresponds to a time dimension (these elementary wave functions are obtained by calculating the eigenfunction of a particle in a one-dimensional box, that is, the edge of the underlying hypercubes). The masses of the electron, of the electron neutrino and of the quark up (first family of fermions) are equal to integer powers of 2 (in  $eV/c^2$ ) [3]. We will show that the fermions of the second and third families are excited states of the fermions of the first family. Indeed, the masses of all elementary fermions follow the formula  $2^n \cdot (p^2)/2$  where  $n$  is an integer [3, 4] calculated for the electron, electron neutrino and quark up and  $p$  is another integer that corresponds to the excited states of the elementary wavefunctions (see Section 3).

## 2. Dimensions of elementary particles

All the theories that aim to understand the elementary particles treat time  $t$  as a special dimension. Thus, many physicists deal with  $n + 1$  space dimensions in particle physics, where the +1 corresponds to the special temporal dimension, thus treated differently. As previously published [3–8], time may be seen as a function of space dimension, if our three-dimensional universe is embedded in a four-dimensional space (due to the positive curvature of our three-dimensional universe).

So, here we will present a simple hypothesis about the classification of elementary particles based on the fact that the space is four-dimensional and that time  $t$  is a dimension like  $x, y$  and  $z$ . Here, this book chapter is dedicated to our hypothesis. This classification is intuitive but next sections of this book chapter, which deal with the masses of the elementary fermions, use and thus demonstrate this classification.

Indeed, with simple arguments, it seems to lead to the Grand Unified Theory (GUT). Time is a function of the fourth dimension of this four-dimensional Euclidean space. If we apply this hypothesis to particle physics, we may say that elementary particles are four-dimensional, three-dimensional and two-dimensional. The coordinates  $(x, y, z, t)$  are not orthonormal. Indeed, time  $t$  evolves as  $\log(r)$  where  $r$  is the co-moving distance in cosmology [6]. Let us make the additional assumption that for each of these four dimensions there are functions like  $\exp(ir_j)$  with  $(r_j = x, y, z, t)$  that vibrate (like in string theory). To find these elementary functions, one has to solve the one-dimensional problem of a particle in a square potential of edge length  $\hbar$  (the Planck constant).

So, our reasoning is simply the description of how to distribute these functions in the four-dimensional space. In the following, the reasoning applies in real space.

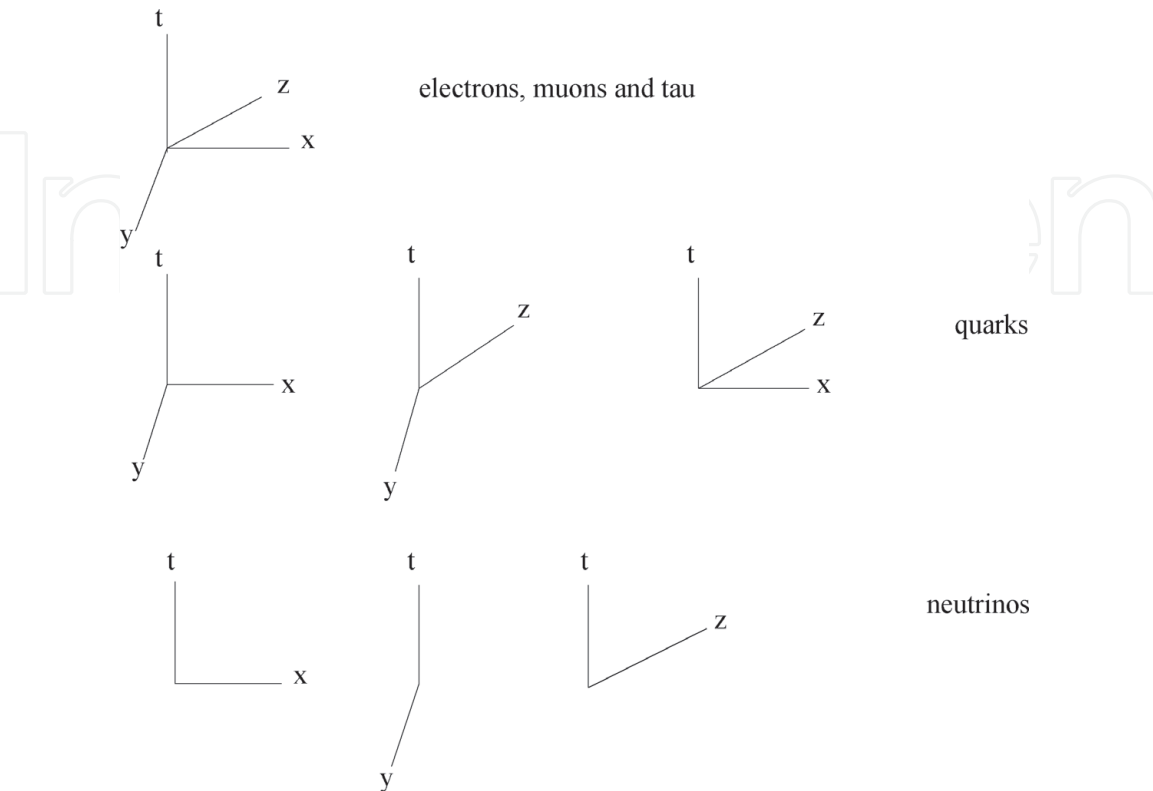
A previous paper of mine (see [3, 9]) predicts that the Higgs potential in real space is a hypercubic box in our four-dimensional space. To obtain the first family of fermions from the Standard Model (i.e., quark up, electron, electron neutrino), one may say that see **Figure 1**:

- the electron is four-dimensional  $(t, x, y, z)$ ;
- the quark up is three-dimensional  $(t, x, y)$  or  $(t, x, z)$  or  $(t, y, z)$ ; and
- the electronic neutrino is two-dimensional  $(t, x)$  and  $x, y$  and  $z$  are equivalent. When this neutrino propagates, there are infinitesimal rotations between the characteristic coordinates (leading to flavor oscillations).

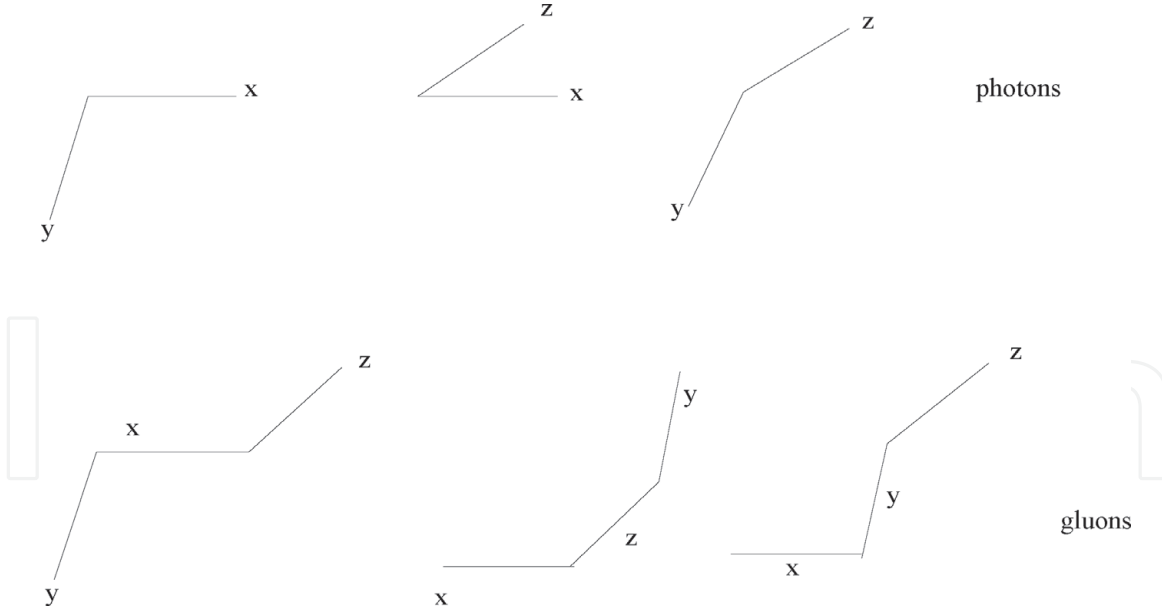
To obtain the masses of the remaining fermions (fermions of the second and third families), one has to add a second quantum number  $p$  (similar to the quantum number obtained for a particle in a square potential of dimensions  $\hbar$ —the Planck length). Thus, the remaining fermions of the Standard Model may be seen as excited states of the first fermion family.

Bosons may be classified with the same assumptions see **Figure 2**:

- the photon is two-dimensional  $(x, y)$  but has no temporal  $t$  coordinate—no mass (indeed with my Higgs potential [3, 9], time at square is proportional to the mass);
- the gluon is three-dimensional  $(x, y, z)$  and has no temporal dimension—no mass (during the strong interaction, one gluon interferes (positive interferences) with two quarks:  $x$  on  $x$ ,  $y$  on  $y$ , etc.);
- the Z and W bosons are three-dimensional with mass  $(t, x, y)$ ; and
- the Higgs boson is four-dimensional  $(x, y, z, t)$ .



**Figure 1.**  
*Dimensions of elementary fermions.*



**Figure 2.**  
*Dimensions of elementary bosons.*

In all these descriptions [5], the geometrical characteristics of elementary particles have been separated from their equation of propagation. With this hypothesis, we obtained a new geometrical classification of elementary particles. Presently, most of the calculations have been made using Feynman graphs, that is, in the space of functions, leading to symmetries that are not yet unified. So in our opinion, the symmetries in the Standard Model do not give the entire description of elementary particles.

In the following section, I will use the geometrical dimensions of the elementary particles to calculate the masses of elementary particles.

### 3. Masses of elementary fermions

In quantum mechanics, the wavefunction gives the most fundamental description of the behavior of a particle; the measurable properties of the particle (such as its position, momentum and energy) may all be derived from the wavefunction. The wavefunction  $\psi(x, t)$  can be found by solving the Schrödinger equation for the system [10].

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t) + V(x, t) \psi(x, t) \quad (1)$$

where  $\hbar$  is the reduced Planck constant,  $m$  is the mass of the particle,  $i$  is the imaginary unit and  $t$  is time. The square potential  $V(x, t)$  is equal to zero for  $x < L$  and  $x > 0$  and for  $t < L$  and  $t > 0$ . We use Von-Karman boundary conditions. Moreover, the domain of definition of the function  $\psi(x, t)$  is  $[0, L]$  for  $x$  and also  $[0, L]$  for  $t$  where  $L$  is the width of the potential  $V(x, t)$ .

The eigenfunctions of the Schrödinger equation may be written:

$$\psi(x, t) = \sqrt{\frac{2}{L}} \exp(-ik_p x) \exp(-i\omega t) \quad (2)$$

where  $\sqrt{\frac{2}{L}}$  normalizes the eigenfunctions. To compute the energy levels of these eigenfunctions, we have:

$$k_p = \frac{p\pi}{L} \quad (3)$$

and

$$E_p = \hbar\omega_p = \frac{p^2\pi^2\hbar^2}{2mL^2}, \quad (4)$$

In order to simplify our calculations, we normalize all constants so that the eigenfunctions are equal to:

$$\psi(x, t) = \sqrt{2} \exp(ix_j) \exp(it) \quad (5)$$

To obtain the masses of all elementary fermions (elementary particles), one has to modify the quantum number  $p$  [4] (similar to the quantum number of a particle in a box). Thus, the remaining fermions of the Standard Model may be seen as excited states of the first fermion family.

Straightforwardly, we make the following hypotheses:

- spacetime has an underlying hypersquare array of edge length  $\hbar$ ;
- elementary wave functions (in  $(x, y, z, t)$  space) are eigenfunctions of a particle in a square potential (reduced parameters)  $\sqrt{2} \exp(-ix)$  for space  $\sqrt{2} \exp(-it)$  for time; and
- the eigenvalues of the elementary wave functions are equal to  $\frac{p^2}{2}$  (with  $p$  an integer number).

In the following subsection, I will use the preceding hypotheses to calculate theoretically the masses of the elementary fermions.

### 3.1 Masses of the electron, muon and tau

The Dirac equation may be written:

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0 \quad (6)$$

with  $\psi$  the wavefunction,  $m$  the mass of the fermion and with the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix}, \quad (7)$$

$$\gamma^1 = \begin{pmatrix} 0 & \sigma_x \\ -\sigma_x & 0 \end{pmatrix}, \quad (8)$$

$$\gamma^2 = \begin{pmatrix} 0 & \sigma_y \\ -\sigma_y & 0 \end{pmatrix}, \quad (9)$$



particle	Theoretical mass ( $eV/c^2$ )	Experimental mass ( $eV/c^2$ ) [11]
electron	$2^{19} = 0.524MeV/c^2$	$0.511MeV/c^2$
muon	$2^{19}.20^2/2 = 104.8MeV/c^2$	$105.6MeV/c^2$
tau	$2^{19}.82^2/2 = 1.76GeV/c^2$	$1.78GeV/c^2$

**Table 1.**

Theoretical and experimental masses of the electron, muon and tau.

$$\gamma^3 = \begin{pmatrix} 0 & \sigma_z \\ -\sigma_z & 0 \end{pmatrix}, \quad (10)$$

where  $\sigma_\nu$  are the Pauli matrices.

Using combinatorial analysis, we obtain Eq. (11) (using the fact that electrons are  $4d$  [5] and that all space dimensions are equivalent).

$$\begin{pmatrix} \gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0 \end{pmatrix} \begin{pmatrix} \partial_1 \Psi \\ \partial_2 \Psi \\ \partial_3 \Psi \\ \partial_1 \Psi \\ \partial_2 \Psi \\ \partial_3 \Psi \\ \partial_1 \Psi \\ \partial_2 \Psi \\ \partial_3 \Psi \\ \partial_0 \Psi \end{pmatrix} = m. \begin{pmatrix} \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \end{pmatrix} \quad (11)$$

There are three possibilities of arranging  $\gamma_1, \gamma_2, \gamma_3$  (the Dirac matrices) over  $x, y$  and  $z$  (all space dimensions are equivalent) and one possibility to arrange  $\sigma_0$  (temporal Pauli matrix: half of  $\gamma_0$ ; because time does not go backward).

The large matrix  $M$  (see Eq. (11)) containing all combinations has a dimension  $9X4 + 2 = 38$ . We see that, with the coordinate vectors  $\sqrt{2} \exp(-it)$  and  $\sqrt{2} \exp(-ix)$  (eigenfunctions of a particle in a square potential), we have to multiply the modified Dirac equation by the Jacobian corresponding to these new coordinates. This Jacobian is equal to  $\sqrt{2}^{38}$  where 38 is the dimension of the large matrix [3]. We multiply the mass of the first particle of this family by the eigenvalues of the eigenfunctions (of the particle).

We decompose the eigenvalues into prime numbers [4]. The number of eigenvalues for the ground state (electron) is 38 (the dimension of the large matrix  $M$ ). For the other particles, we take into account the spinor  $(1, 0)^T$  corresponding to the  $\sigma_0$  Pauli matrix. So except for the electron, there are 37 eigenvalues for each particle [4].

- The mass of the electron is equal to  $\sqrt{2}^{38} = 2^{19} eV/c^2 = 2^{19} \cdot \left(\frac{1^2}{2}\right)^{19} \cdot \left(\frac{2^2}{2}\right)^{19} = 0.524MeV/c^2 \approx 0.511 MeV/c^2$ .
- The mass of the muon is equal to  $2^{19}.20^2/2 = 2^{19} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{5^2}{2} \cdot \left(\frac{1^2}{2}\right)^{16} \cdot \left(\frac{2^2}{2}\right)^{16} = 104.8MeV/c^2 \approx 105.6 MeV/c^2$ .

- The mass of the tau is equal to  $2^{19} \cdot 82^2 / 2 = 2^{19} \cdot \frac{41^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \left(\frac{1^2}{2}\right)^{17} \cdot \left(\frac{2^2}{2}\right)^{17} = 1.76 \text{ GeV}/c^2 \approx 1.78 \text{ GeV}/c^2$ .

The values in italic are the experimental masses [11].

We see that for the tau particle, one of the eigenvalues  $\left(\frac{41^2}{2}\right)$  is much larger than the others. This may explain the short lifetime of this particle.

The masses (theoretical and experimental) of the electron, muon and tau are summarized in **Table 1**.

### 3.2 Masses of the quarks

For quarks, we have

$$i\gamma^\mu \partial_\mu \psi = m\psi \tag{12}$$

The Dirac matrices are representative of infinitesimal rotations within the wavefunction of a given elementary particle.

Using combinatorial analysis, we obtain Eq. (13) (using the fact that quarks are  $3d$  [5] and that all space dimensions are equivalent). There are three possibilities for arranging  $\gamma_1, \gamma_2, \gamma_3$  (the Dirac matrices) over  $x, y$  and  $z$  (all space dimensions are equivalent). There is one possibility to arrange  $\sigma_0$  (temporal Pauli matrix; half of  $\gamma_0$ , because time does not go backward) for each combination of spatial Dirac matrices ( $x, y$ ;  $x, z$ ; and  $y, z$ ). We have to take into account that the quarks are three-dimensional. So, the matrix  $M$  containing all combinations has a dimension equal to  $9 \times 4 + 3 \times 2 = 42$ .

$$\begin{pmatrix} \gamma_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_0 \end{pmatrix} \begin{pmatrix} \partial_1 \Psi \\ \partial_2 \Psi \\ \partial_3 \Psi \\ \partial_0 \Psi \\ \partial_1 \Psi \\ \partial_2 \Psi \\ \partial_3 \Psi \\ \partial_0 \Psi \\ \partial_1 \Psi \\ \partial_2 \Psi \\ \partial_3 \Psi \\ \partial_0 \Psi \end{pmatrix} = m \begin{pmatrix} \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \\ \Psi \end{pmatrix} \tag{13}$$

We see that, with the coordinate vectors  $\sqrt{2} \exp(-it)$  and  $\sqrt{2} \exp(-ix)$  (eigenfunctions of the underlying hypersquare array), we have to multiply the modified Dirac equation by the Jacobian corresponding to these new coordinates. This Jacobian is equal to  $\sqrt{2}^{42}$  where 42 is the dimension of the matrix [3]. We multiply the mass of the first particle of the quarks family by the eigenvalues of the eigenfunctions (of the particle). We decompose the eigenvalues into prime numbers [4]. The number of eigenvalues for the ground state (quark up) is 42



(the dimension of the large matrix  $M$ , see Eq. (13)). For the other quarks, we take into account the spinor  $(1, 0)^T$  corresponding to the three  $\sigma_0$  Pauli matrices. So except for the quark up, there are 39 eigenvalues for each quark [4].

- The quark up has a mass equal to  $\sqrt{2}^{42} = 2^{21} eV/c^2 = 2^{21} \cdot \left(\frac{1^2}{2}\right)^{21} \cdot \left(\frac{2^2}{2}\right)^{21} = 2.09 MeV/c^2 \approx 2.2 MeV/c^2$ .
- The quark down has a mass equal to  $2^{21} \cdot \frac{2^2}{2} = 2^{21} \cdot \frac{2^2}{2} \cdot \left(\frac{1^2}{2}\right)^{19} \cdot \left(\frac{2^2}{2}\right)^{19} = 4.19 MeV/c^2 \approx 4.7 MeV/c^2$ .
- The quark strange has a mass equal to  $2^{21} \cdot \frac{9^2}{2} = 2^{21} \cdot \frac{3^2}{2} \cdot \frac{3^2}{2} \cdot \frac{2^2}{2} \cdot \left(\frac{1^2}{2}\right)^{18} \cdot \left(\frac{2^2}{2}\right)^{18} = 84.9 MeV/c^2 \approx 96 MeV/c^2$ .
- The quark charm has a mass equal to  $2^{21} \cdot \frac{36^2}{2} = 2^{21} \cdot \frac{3^2}{2} \cdot \frac{3^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \left(\frac{1^2}{2}\right)^{16} \cdot \left(\frac{2^2}{2}\right)^{16} = 1.35 GeV/c^2 \approx 1.27 GeV/c^2$ .
- The quark bottom has a mass equal to  $2^{21} \cdot \frac{63^2}{2} = 2^{21} \cdot \frac{3^2}{2} \cdot \frac{3^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{7^2}{2} \cdot \left(\frac{1^2}{2}\right)^{17} \cdot \left(\frac{2^2}{2}\right)^{17} = 4.16 GeV/c^2 \approx 4.18 GeV/c^2$ .
- The quark top has a mass equal to  $2^{21} \cdot \frac{405^2}{2} = 2^{21} \cdot \frac{3^2}{2} \cdot \frac{3^2}{2} \cdot \frac{3^2}{2} \cdot \frac{3^2}{2} \cdot \frac{5^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \frac{2^2}{2} \cdot \left(\frac{1^2}{2}\right)^{15} \cdot \left(\frac{2^2}{2}\right)^{15} = 171.9 GeV/c^2 \approx 173 GeV/c^2$ .

The values in italic are the experimental masses [11].  
The theoretical and experimental masses of the quarks family are summarized in **Table 2**.

quark	Theoretical mass ( $eV/c^2$ )	Experimental mass ( $eV/c^2$ ) [11]
up	$2^{21} = 2.09 MeV/c^2$	$2.2 MeV/c^2$
down	$2^{21} \cdot 2^2 / 2 = 4.19 MeV/c^2$	$4.7 MeV/c^2$
strange	$2^{21} \cdot 9^2 / 2 = 84.9 MeV/c^2$	$96 MeV/c^2$
charm	$2^{21} \cdot 36^2 / 2 = 1.35 GeV/c^2$	$1.27 GeV/c^2$
bottom	$2^{21} \cdot 63^2 / 2 = 4.16 GeV/c^2$	$4.18 GeV/c^2$
top	$2^{21} \cdot 405^2 / 2 = 171.9 GeV/c^2$	$173 GeV/c^2$

**Table 2.**  
*Theoretical and experimental masses of the quarks family.*

particle	Theoretical mass	Experimental mass: upper limit [11]
electron neutrino	$2 eV c^2$	$2.5 eV c^2$
muon neutrino	$412^2 = 169.7 keV c^2$	$170 keV c^2$
tau neutrino	$3937^2 = 15.4 MeV c^2$	$18 MeV c^2$

**Table 3.**  
*Theoretical masses of the neutrinos and upper limits of experimental masses.*

### 3.3 Masses of the neutrinos

Up to now, there is no theoretical propagation equation for the neutrinos.

If we use the eigenvalues of the elementary wave functions like for quarks and electrons, muons and taus, we may write [4]:

- the mass of the electron neutrino is equal to  $2eV/c^2$ ;
- the mass of the muon neutrino is equal to  $2 \cdot \frac{412^2}{2} = 412^2 eV/c^2 = 169 keV/c^2$ ; and
- the mass of the tau neutrino is equal to  $2 \cdot \frac{3937^2}{2} = 3937^2 eV/c^2 = 15.4 MeV/c^2$ .

Hence, we found theoretical values of the masses of the neutrinos, which are in good agreement with the experimental masses (**Table 3**).

## 4. Conclusion


In this chapter, the calculations of the masses of all the known elementary fermions are made in real space. At the beginning of this book chapter (Section 2), I presented a classification of elementary particles over all space and temporal dimensions. Using this geometrical classification (which is intuitive), we found the theoretical values of masses for all the elementary fermions (electrons, muons and taus; all quarks and all neutrinos). The theoretical masses are in good agreement with the experimental masses (the differences between theoretical and experimental masses are less than 10% except for the quarks down and strange). To conclude, our theory unifies all elementary fermions: we use the same approach to all these fermions (geometry and the underlying hypersquare array of spacetime). In the future, there is a possibility to analyze the symmetries of these particles and compare them to the symmetries of the Standard Model.

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